A Joint-Chirp-rate-Time-Frequency Transform for Binary Black Hole Merger Signal Detection using Spectrograms

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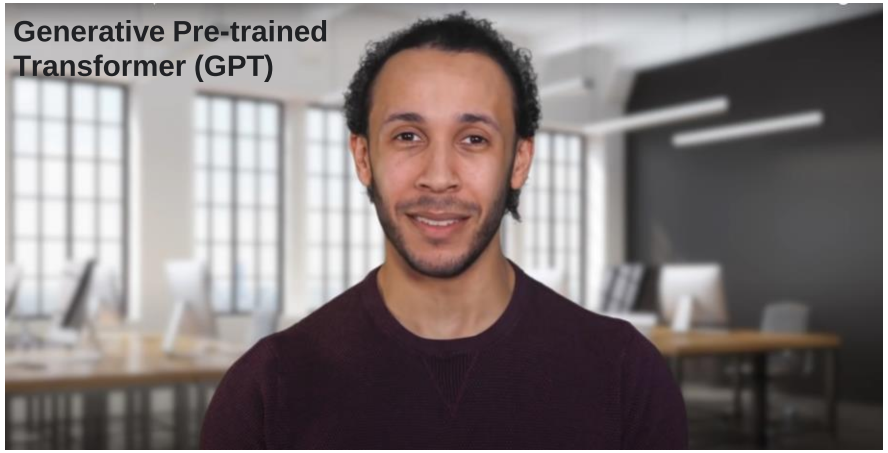
Motivation

- Machine Learning (ML) algorithms have transformed the methods of data analysis, image pattern recognition, and math modeling.
- Artificial Neural Networks (ANNs) are among the most talked about techniques in the ML family with a wide range of applications.
  - Applications of ANNs

Self-driving Cars*

*Side-by-side camera view and ANN annotated LIDAR data (Waymo)



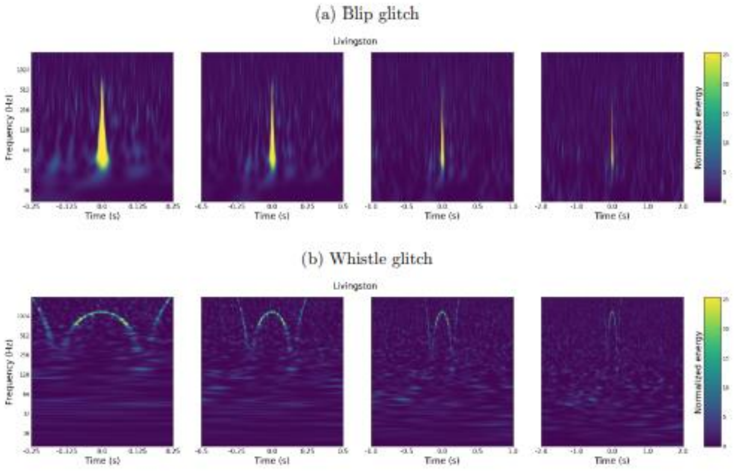
Natural Language Processing^

^AI chat robot with facial expressions, movements, and voice generated using GPT-3 (OpenAI)

What can ANNs do to accelerate Gravitational Wave (GW) research?

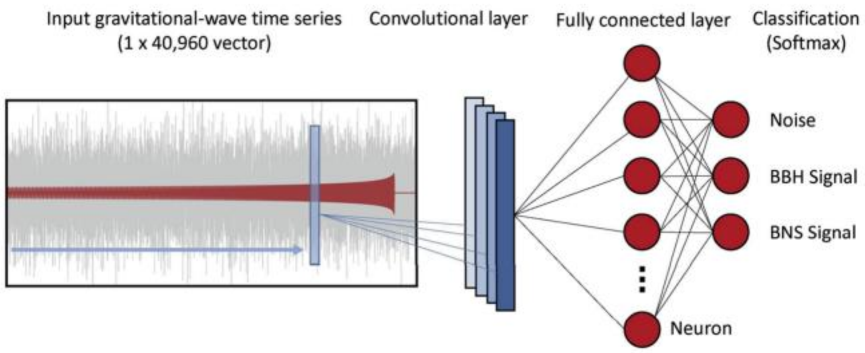
Motivation

- Active research areas:
  - Detector transient noise (glitch) classification.
  - Real-time Binary Black Hole (BBH) Binary Neutron Star (BNS) merger event detection.
  - BBH/BNS merger event forecasting.



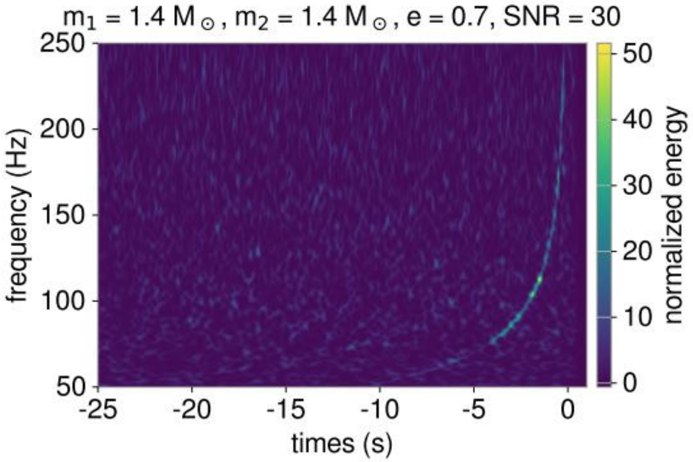
Glitch Classification*

*Spectrograms** of two types of glitches (Gravity Spy)



BBH/BNS Merger Detection^

^ Convolutional Neural Network (CNN) in merger event detection, classification. (Plamen G. Krastev)



BBH/BNS Merger Forecasting◦

◦ **Spectrogram** of a simulated BNS merger signal (W. Wei and et al.)

M. Zevin and et al., "Gravity spy: integrating advanced ligo detector characterization, machine learning, and citizen science," Classical and Quantum Gravity, vol. 34, no. 6, p. 064003, 2017

Krastev, P. G. (2020). Real-time detection of gravitational waves from binary neutron stars using Artificial Neural Networks. *Physics Letters B*, *803*, 135330.

W. Wei and et al., "Deep learning with quantized neural networks for gravitational-wave forecasting of eccentric compact binary coalescence," The Astrophysical Journal, vol. 919, no. 2, p. 82, 2021

Objective

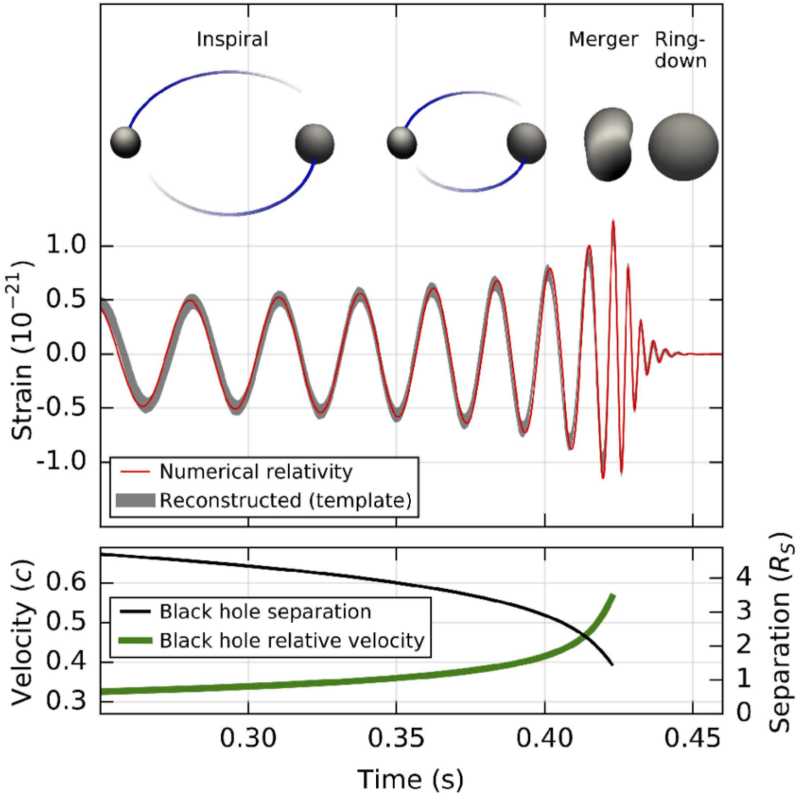
- Design a transform method that produces **chirp-rate enhanced spectrograms** to improve spectrogram classification networks' performance in low signal-to-noise ratio BBH, BNS merger signal detection and forecasting.

Current Detection Techniques

BBH Chirp Signal Detection

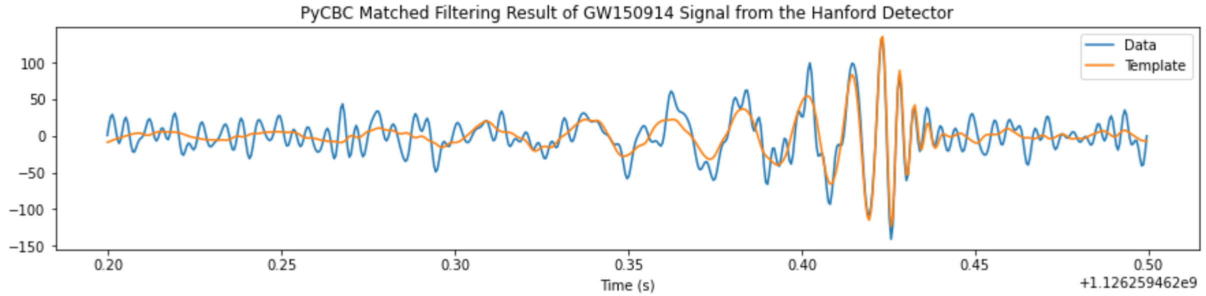
Chirp signal: ~ changing frequency

BBH Merger Process and Waveform



Francisco R. Villatoro (2018)

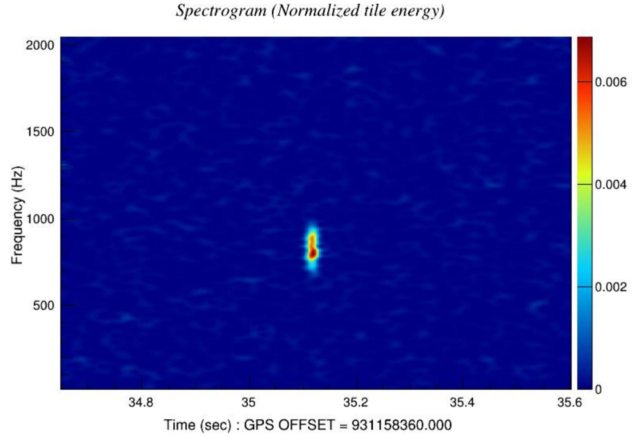
Technique 1: **Templated Search** – Matched Filtering



Hanford detector signal of BBH merger event GW150914 (September 14 2015, 09:50:45 UTC) plotted against the matched waveform template in PyCBC.

Technique 2: **Non-templated Search** – Burst Search

Spectrogram Generation



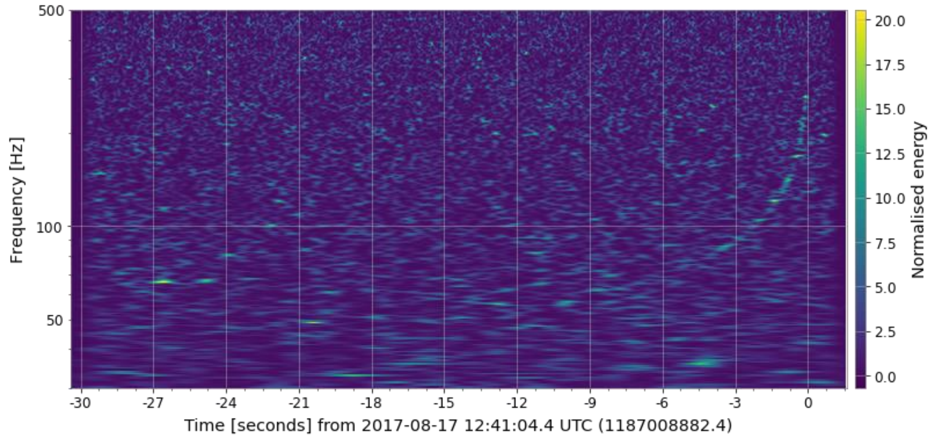
The spectrogram of an unknown event recorded by the Livingston detector at GPS time 931158360 (July 8 2009, 07:05:45 UTC), generated by the coherence waveBurst pipeline.

Existing Spectrogram Generation Methods

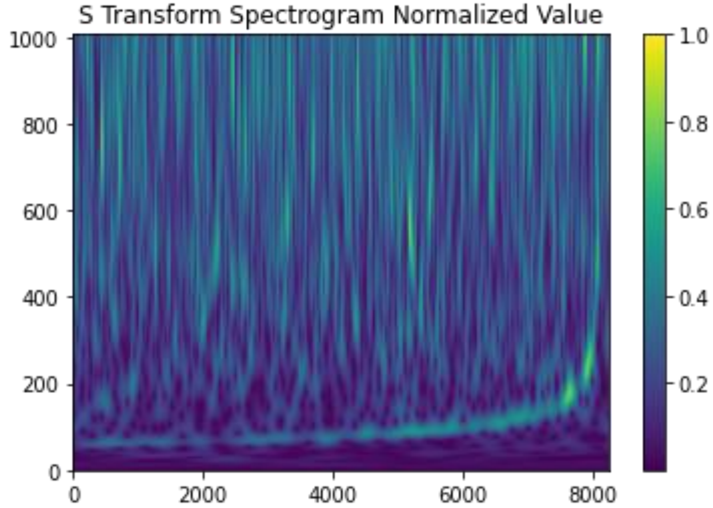
- Short-time Fourier Transform (STFT)
- Gabor Transform (GT)
- Constant Q Transform (CQT)
- S (Stockwell) Transform (ST)
- …

- All use the Fourier transform as the foundation.
- Only decompose the relationship between time and frequency.
- The defining characteristic of a BBH merger signal, **the chirp**, is abandoned.

Constant Q Transform of GW 150914



S Transform of a simulated BBH merger



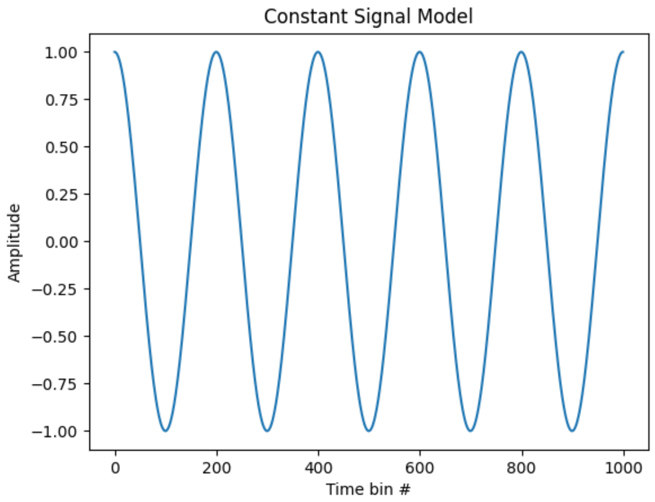
Simulated merger m1 = m2 =, normalized amplitude 1, injected to Gaussian noise of amplitude 10.

Obtaining the Chirp-rate Information

Fourier Transform (FT)

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi\Omega t} dt,$$

Matching the input signal x(t) to a **constant frequency** signal model: $e^{-\mathrm{i}2\pi\Omega t}$.

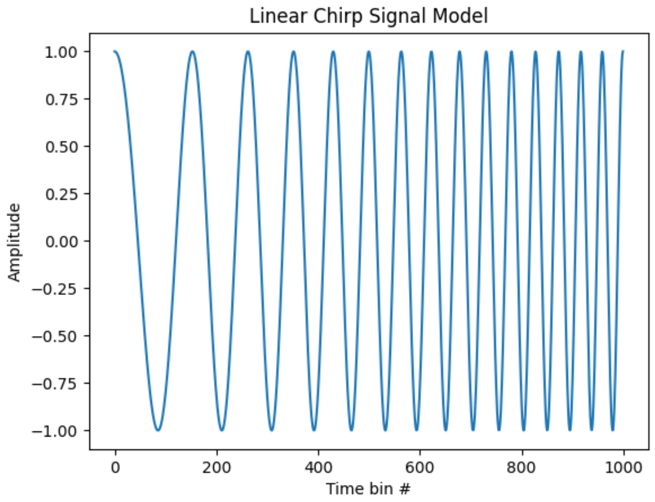


Linear Chirp Transform (LCT)

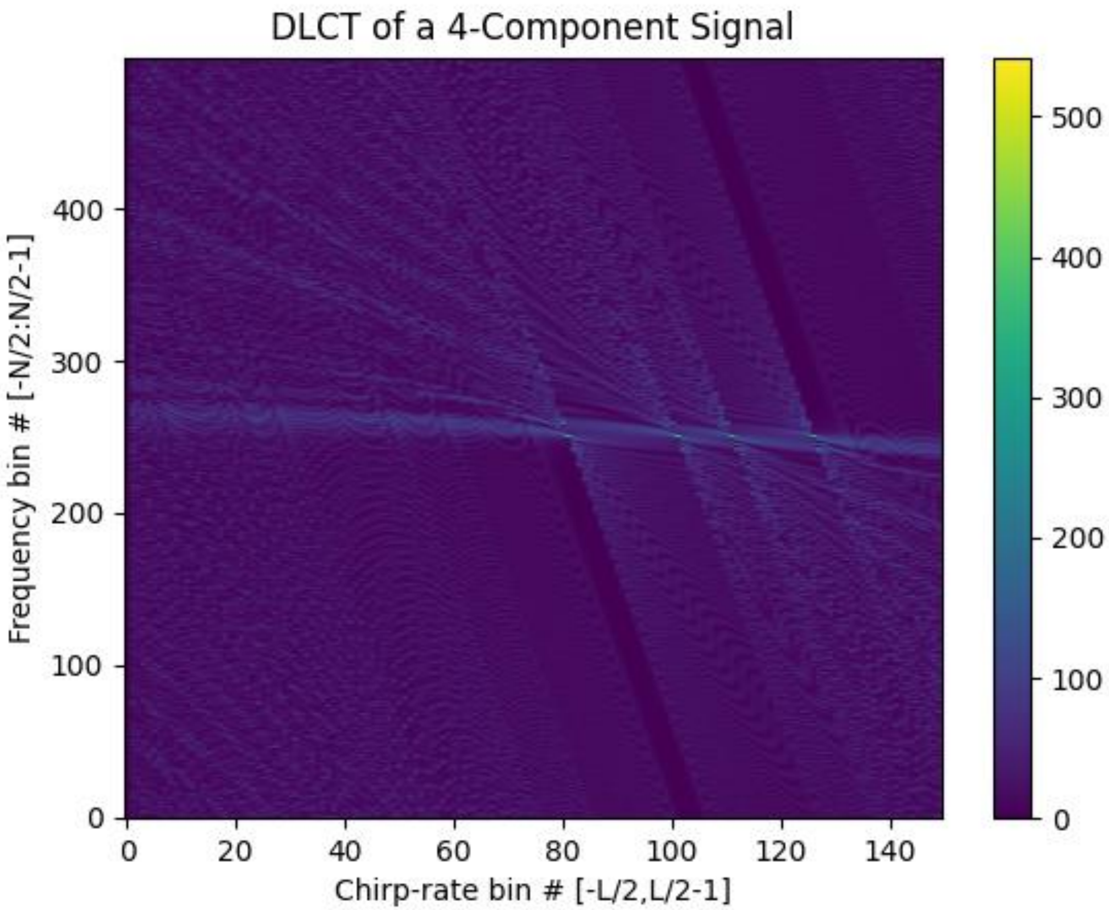
$$X(\Omega, \gamma) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi(\Omega t + \gamma t^2)} dt.$$

O, A, Alkaishriwo & L.F. Chaparro (2012)

Matching the input signal x(t) to a **linear chirp** signal model: $e^{-\mathrm{i}2\pi(\Omega t+\gamma t^2)}$.



Obtaining the Chirp-rate Information



Discrete Linear Chirp Transform (DLCT) of a 4-component linear chirp signal.

Linear Chirp Transform (LCT)

$$X(\Omega,\gamma)=\int_{-\infty}^{\infty}x(t)e^{-i2\pi(\Omega t+\gamma t^2)}dt.$$

O, A, Alkaishriwo & L.F. Chaparro (2012)

$$h_{4-component}(t)= e^{j2\pi t^2}+e^{j2\pi 5t^2}+e^{j2\pi 7t^2}+e^{j2\pi 10t^2}$$

One can obtain the chirp rate and starting frequency of each chirp signal by further processing the Linear Chirp Transform frequency-chirp-rate diagram.

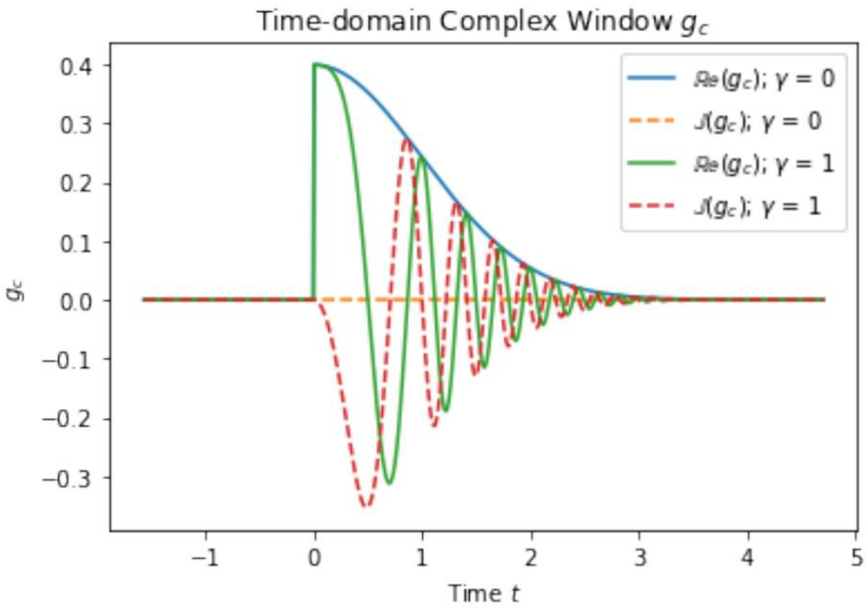
The Joint-Chirp-Rate-Time-Frequency Transform (JCTFT)

$$H_J(\gamma, \tau, \Omega) = \int_{-\infty}^{\infty} h(t) g_c(\gamma, t-\tau, \Omega) e^{-i2\pi\Omega t} dt,$$

$\xrightarrow{\gamma = 0}$ Short-Time Fourier Transform

$$g_c(\gamma, t-\tau, \Omega) = \frac{|\Omega_0 + \mu\Omega|}{\sqrt{2\pi}} \frac{1 + \frac{t-\tau}{|t-\tau|}}{2} e^{-(t-\tau)^2\left[\frac{(\Omega_0+\mu\Omega)^2}{2} + i2\pi\gamma\right]},$$

$\xrightarrow{\gamma = 0}$ Gaussian Window



A 2D representation by taking an orthogonal projection of JCTFT along the chirp rate axis:

$$\boxed{S_J(\Omega, \tau) = \int_{-L}^{L} H_J(\gamma, \tau, \Omega) d\gamma,}$$

An Alternative Definition using the Convolution Theorem

$$\begin{aligned} H_J(\gamma,\tau,\Omega) &= \mathcal{F}\{f(t)g_c(t-\tau,\gamma,\Omega)\} \\ &= \mathcal{F}\{f(t)\} * \mathcal{F}\{g_c(t-\tau,\gamma,\Omega)\} \\ &= \mathcal{F}\{f(t)\} * \mathcal{F}\{g_c(t,\gamma,\Omega)\}e^{-i2\pi\Omega\tau}, \end{aligned}$$

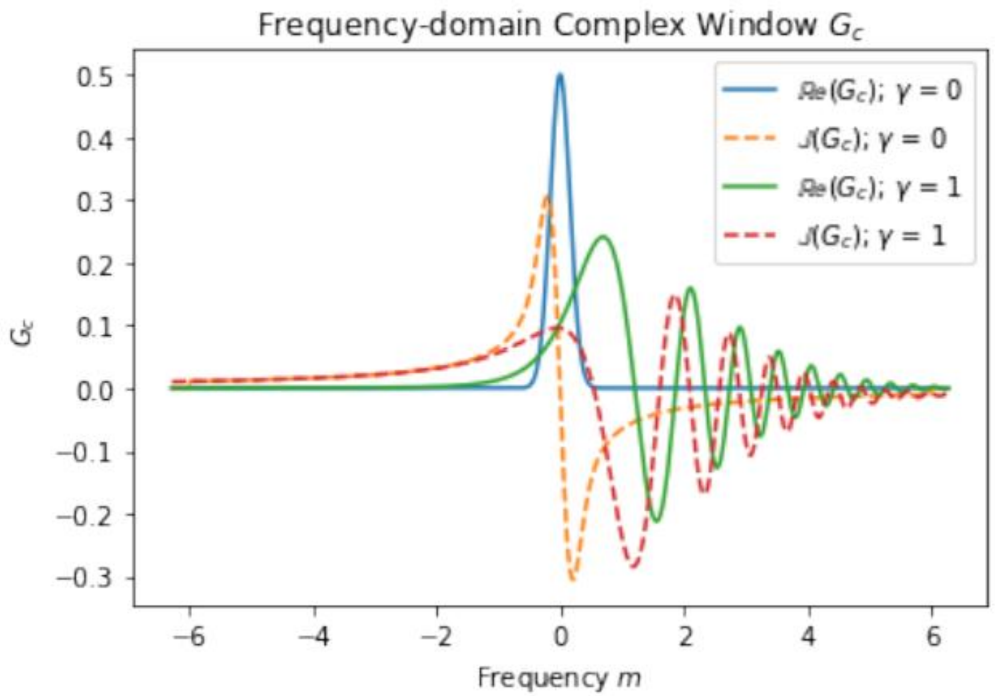
an alternative frequency domain representation of the JCTFT is

$$\boxed{H_J(\gamma,\tau,\Omega) = \int_{-\infty}^{\infty} H(\Omega+\alpha)G_c(\gamma,\Omega,\alpha)e^{i2\pi\alpha\tau}d\alpha,}$$

and Gc is the Fourier transform of the complex linear chirp window function $g_c(t)$ with a dummy frequency variable α due to the convolution:

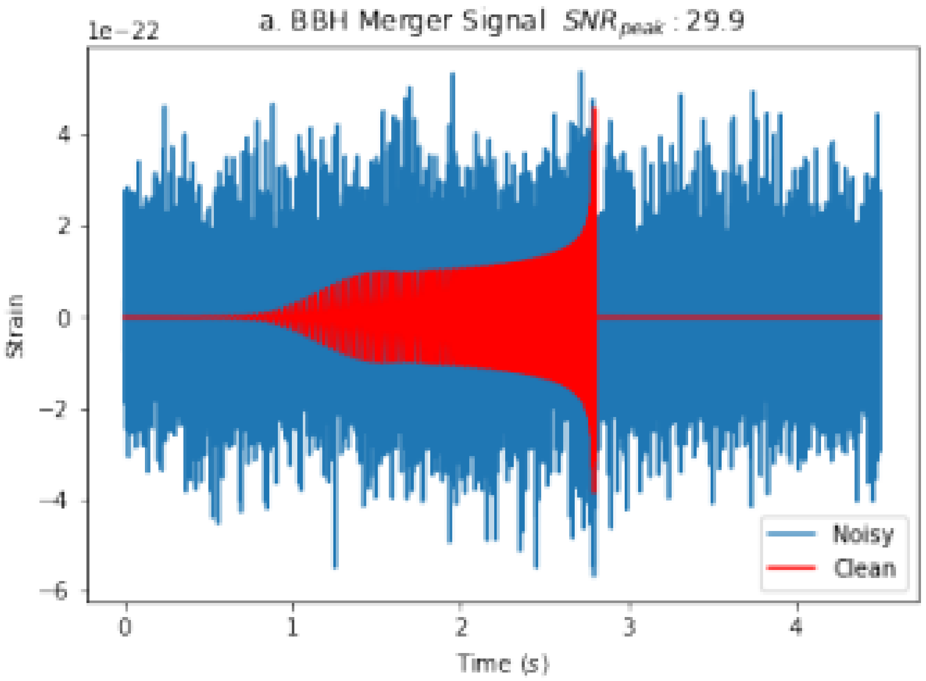
$$G_c(\gamma,\Omega,\alpha) = \frac{|\Omega_0+\mu\Omega|}{\sqrt{2\pi}}\{e^{-\pi^2\alpha^2/z}(\frac{1}{2}\sqrt{\frac{\pi}{z}}[1-erf(\frac{i\pi\alpha}{\sqrt{z}})])\},$$

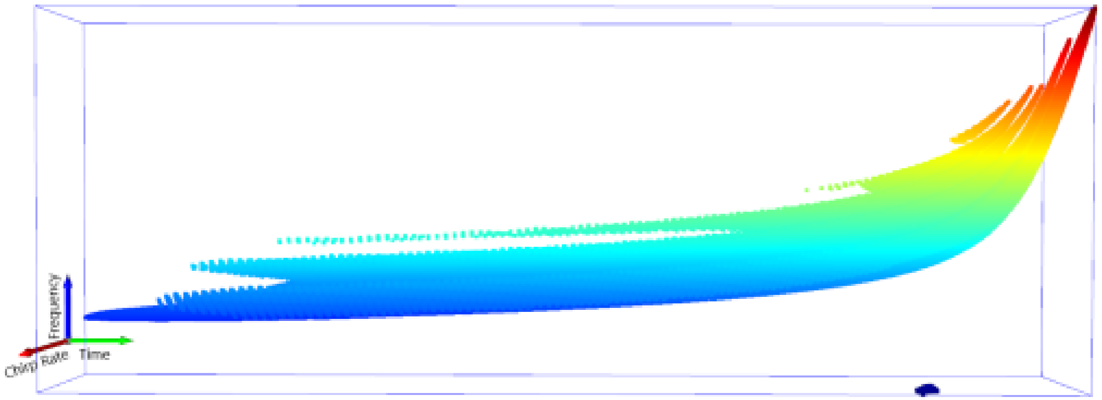
where $z = a+ib$, $a = \frac{(\Omega_0+\mu\Omega)^2}{2}$, and $b = 2\pi\gamma$.



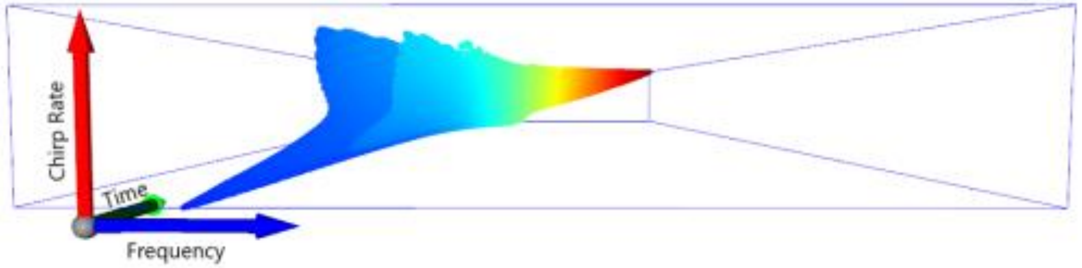
The Joint-Chirp-Rate-Time-Frequency Transform (JCTFT)

$$H_J(\gamma,\tau,\Omega) = \int_{-\infty}^{\infty} H(\Omega+\alpha)G_c(\gamma,\Omega,\alpha)e^{i2\pi\alpha\tau}d\alpha,$$





- 3D JCTFT ***Frequency vs. Time Front***



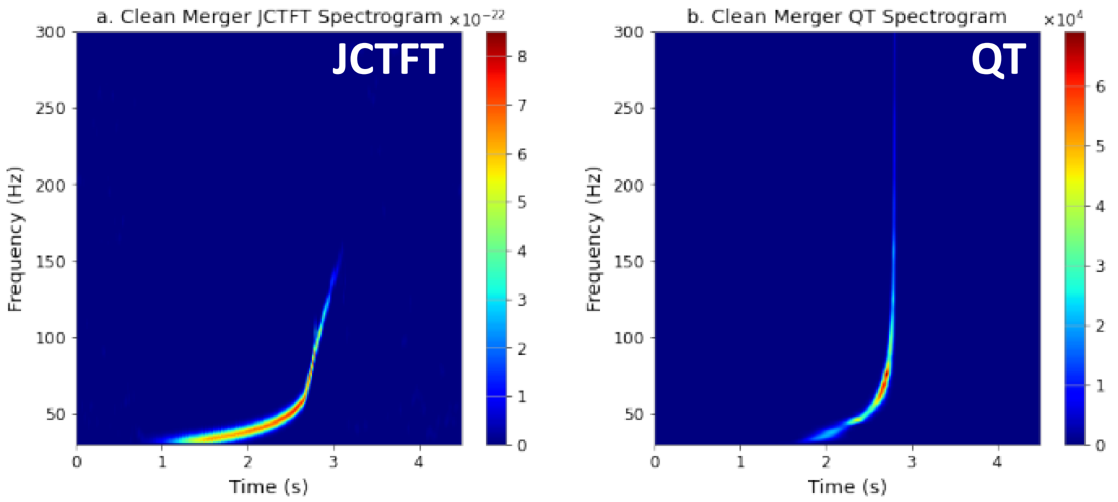
- 3D JCTFT ***Chirp-rate vs. Frequency Front***

Test signal: ***Merger***: *SEOBNR-Phenom Model*
Starting Freq: 30 Hz ; Masses: 15 Solar, 10 Solar

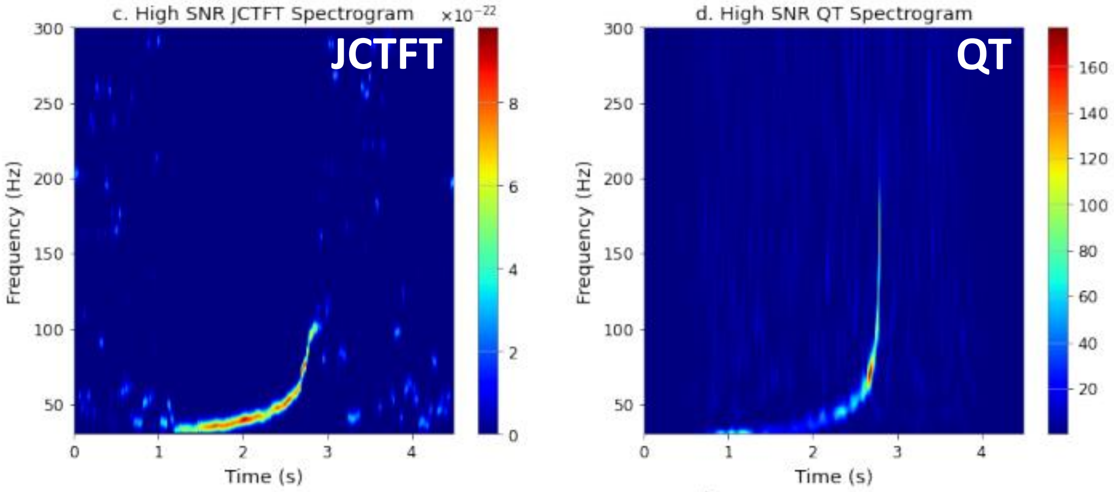
The Joint-Chirp-Rate-Time-Frequency Transform (JCTFT)

$$H_J(\gamma,\tau,\Omega) = \int_{-\infty}^{\infty} H(\Omega+\alpha) G_c(\gamma,\Omega,\alpha) e^{i2\pi\alpha\tau} d\alpha,$$

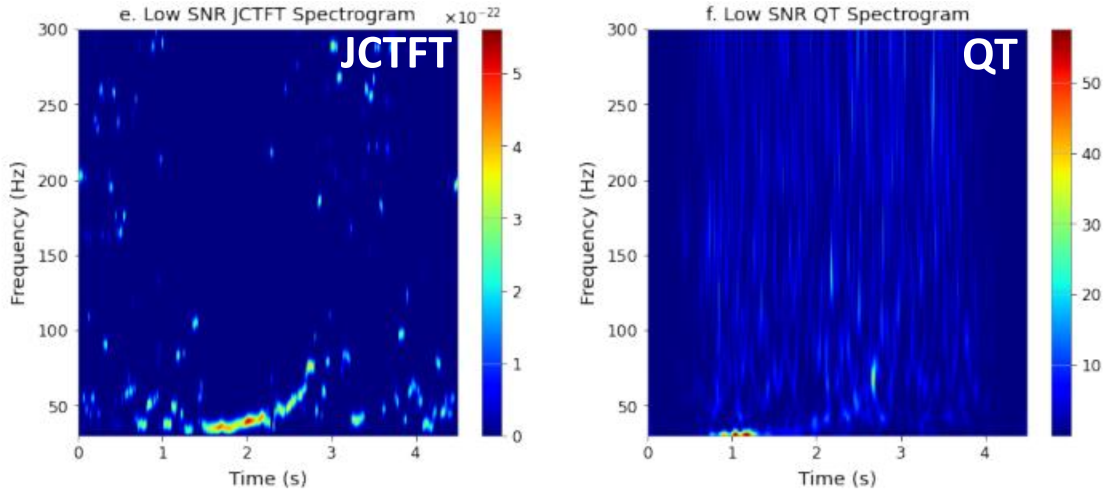
$$\boxed{S_J(\Omega,\tau) = \int_{-L}^{L} H_J(\gamma,\tau,\Omega) d\gamma,}$$



- ***Clean Merger***



- ***Noisy Merger SNR 29.9***

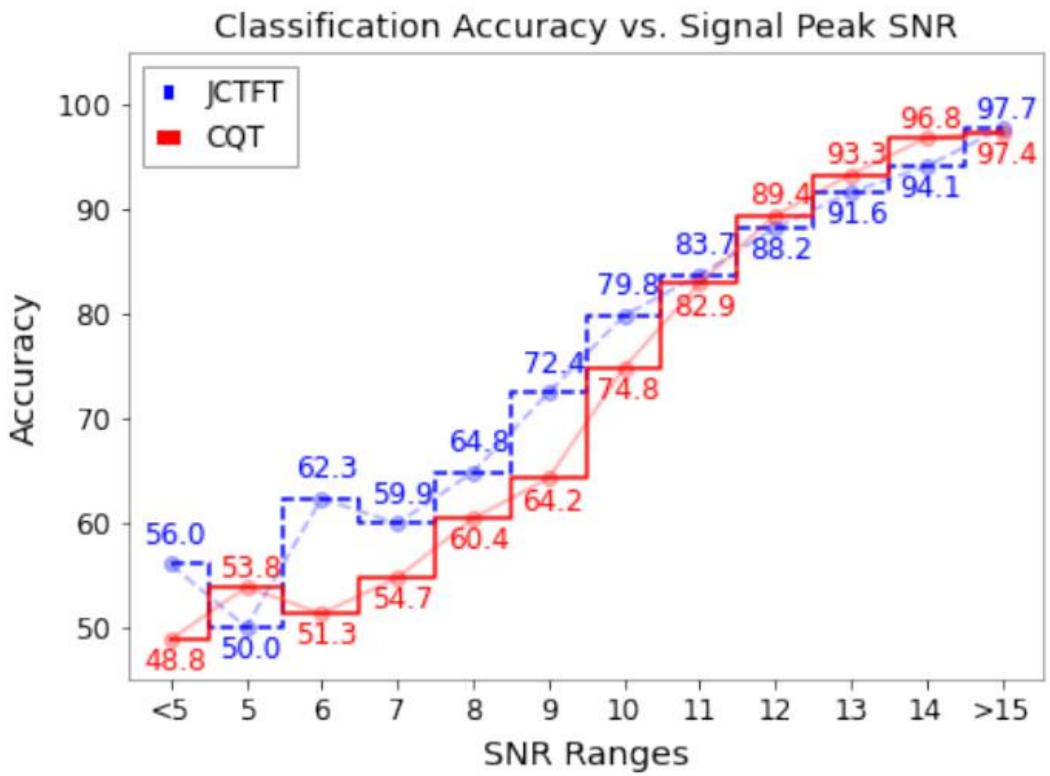


- ***Noisy Merger SNR 10.4***

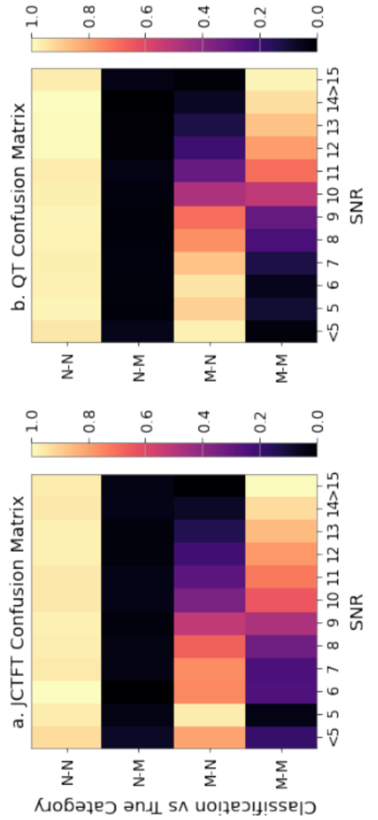
Test signal: ***Merger**: SEOBNR-Phenom Model*
Starting Freq: 30 Hz ; Masses: 15 Solar, 10 Solar

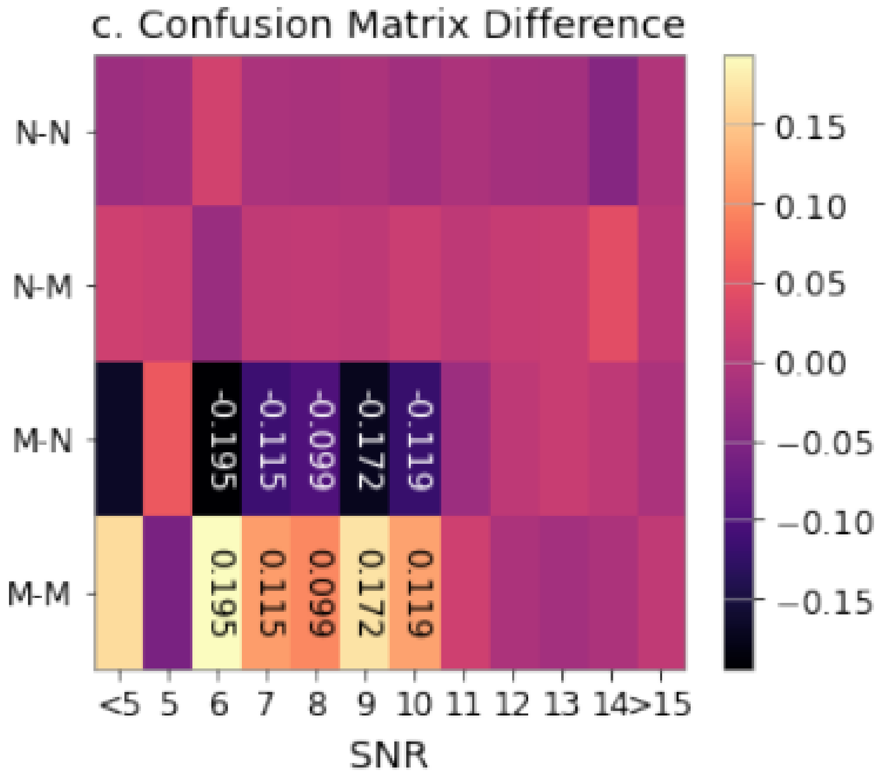
***Noise**: aLIGO 0-detuned-Higher-Power*

Classification Accuracy using Inception V3 Network



*SNR-categorized **classification accuracy** of the JCTFT and QT-trained Inception merger detection network.*





*SNR-categorized **confusion matrix** of the JCTFT and QT-trained Inception merger detection network.*

*On average **14% better performance** for simulated BBH merger signals with SNR 6-10.

Conclusions and Future Work

Conclusions:

- The JCTFT decomposes time-series signals into chirp rate, time and frequency, and establishes the relationship between the three quantities.
- Improved neural network classification performance using simulated BBH merger signals with SNR 6-10.
- The JCTFT and methods extended from it pave the way for new three-dimensional chirp signal search and analysis techniques, using either classical methods or machine learning algorithms.

Next,

- An improved chirp signal peak detection algorithm for more accurate chirp-rate estimation.
- Probe the potential of detector glitch classification and analysis using the JCTFT.
- Investigate the JCTFT periodicity and signal peak characteristics.
- Investigate the effects of these methods in low-latency ANN BBH merger detection pipelines.

Our Team